Sustainability of public debt :
the impact of real long-run determinants of r – g

Freddy Heylen

Dept. of Economics, Ghent University

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Content

1. Public debt dynamics and sustainability: $r - g$ as a key determinant
   - main mechanisms and a first look at data
   - important qualifications and ‘what about the future?’
2. Theory and empirical research on $r - g$
3. Projections for future $r - g$ and related debt dynamics
4. Conclusions, discussion

1. Public debt dynamics: $r - g$ as a key determinant

- Basics... the equation for the dynamics of public debt (*)

$$b_t = \frac{B_t}{Y_t} = \frac{B_{t-1}(1 + r_t) + PDEF_t}{Y_{t-1}(1 + g_t)}$$

$$\Delta b_t = pdef_t + \frac{(r_t - g_t)}{(1+g_t)} b_{t-1} \qquad (1)$$

b_t : public debt in percent of nominal GDP at the end of year t

B_t: outstanding public debt (nominal) at the end of year t

Y_t : nominal GDP in year t

r_t : implicit nominal interest rate on outstanding public debt in year t (%)

g_t : growth rate of nominal GDP in year t (%)

$PDEF_t$: primary budget deficit in year t ($pdef_t = PDEF_t / Y_t$)

(*) Assumption: no 'stock-flow adjustments'.

1. Public debt dynamics: $r - g$ as a key determinant

- Basics... the equation for the dynamics of public debt

$$b_t = \frac{B_t}{Y_t} = \frac{B_{t-1}(1 + r_t) + PDEF_t}{Y_{t-1}(1 + g_t)}$$

$$\Delta b_t = pdef_t + \frac{(r_t - g_t)}{(1+g_t)} b_{t-1} \qquad (1)$$

- Condition for stabilisation of the public debt ratio (Δb_t=0) ?

$$pdef_t = -\frac{(r_t - g_t)}{(1+g_t)} b_{t-1}$$

When $r_t > g_t$: a primary surplus (negative deficit) is necessary

$$pdef_t = -\frac{(r_t - g_t)}{(1+g_t)} b_{t-1} < 0$$ required primary surplus

1. Public debt dynamics: $r - g$ as a key determinant

➢ Condition for stabilisation of the public debt ratio (Δb_t=0) ?

When $r_t < g_t$: public debt stabilisation is consistent with a primary deficit

$$pdef_t = -\frac{(r_t - g_t)}{(1+g_t)} b_{t-1} > 0$$ allowed primary deficit

Possible outcomes for the evolution of public debt?

- Decreasing debt ratio (when the actual primary deficit < allowed)
- Rising debt ratio (when the actual primary deficit > allowed)

With a stable $pdef$ (=$pdef^*$), r (=r^*) and g (=g^*), the public debt ratio convergences to an equilibrium level, whatever the starting point:

$$b^* = \frac{pdef^*}{(g^* - r^*)/(1+g^*)}$$

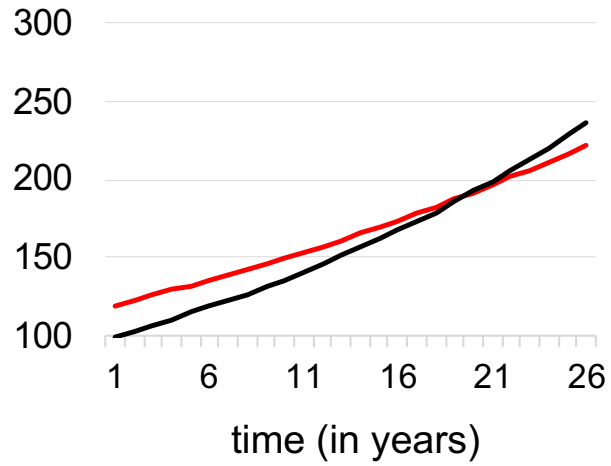
- Temporary (high) primary deficits and shocks to public debt : no fiscal cost, provided r and g are not adversely affected.

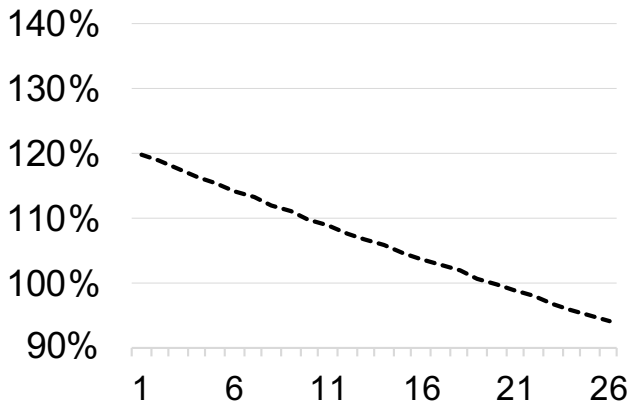
Illustration: Initial situation: Y = 100, B = 120, b = 120%, r = 2,5%, g = 3,5%

→ allowed primary deficit: 1.16%

(a)

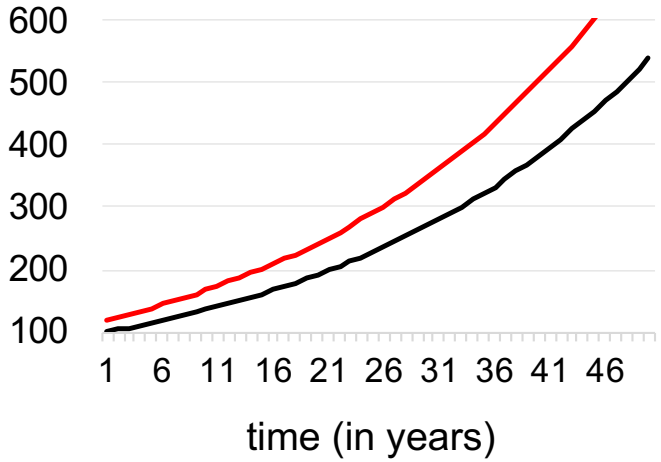
$pdef = 0\%$ in all years

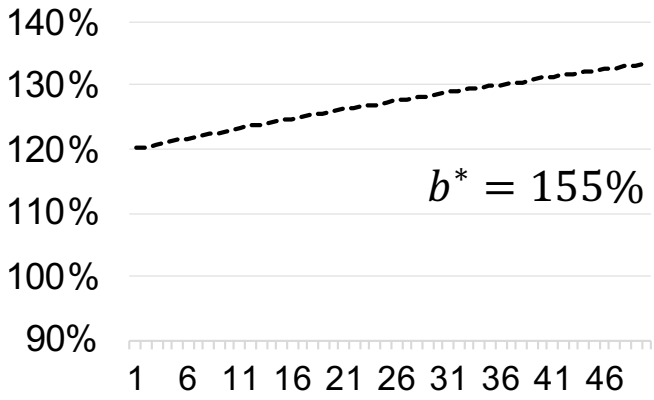




(b)

$pdef = 1.5\%$ in all years

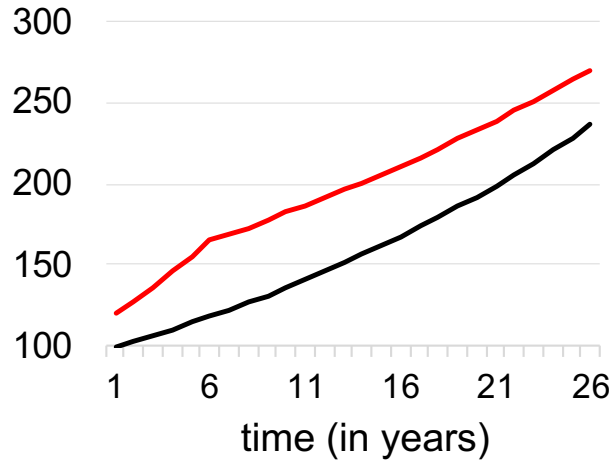


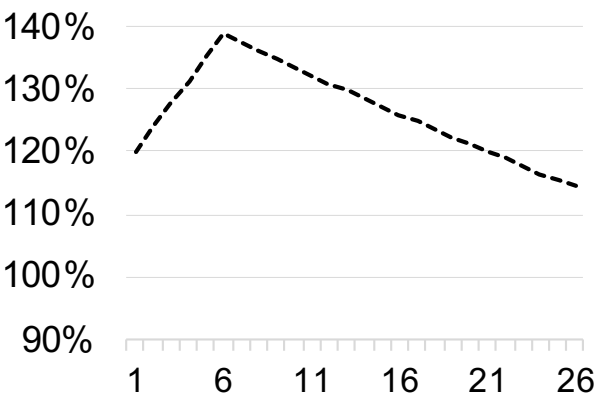


(c)

$pdef = 5\%$ in years 1 – 5

$pdef = 0\%$ from year 6

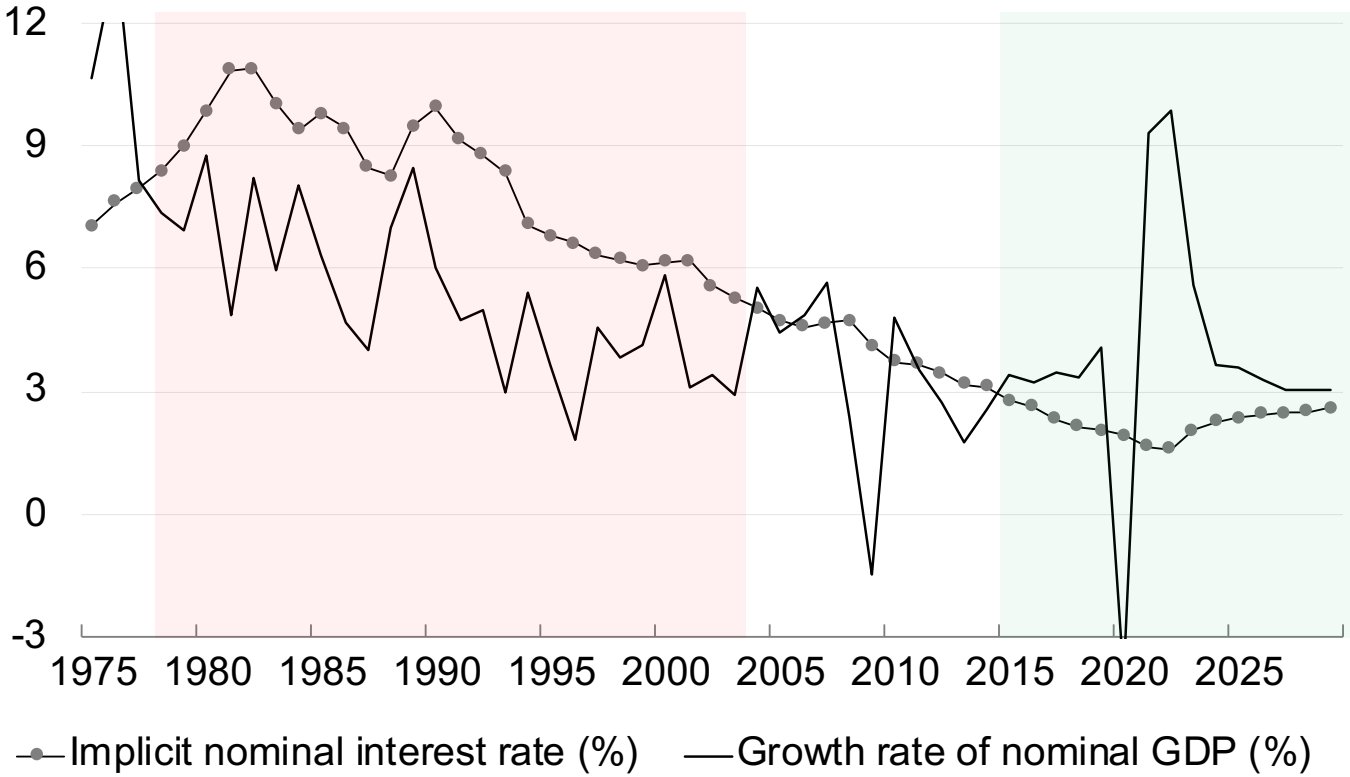




Public debt (B_t) — nom. GDP (Y_t) - - - Public debt ratio (b_t)

1. Public debt dynamics: $r - g$, first look at data

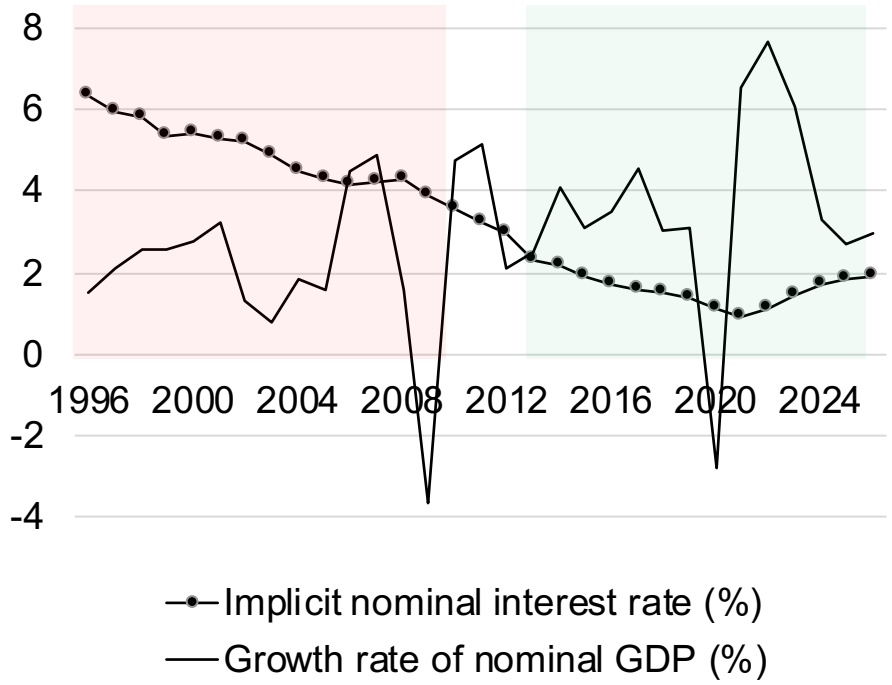
Belgium: 1975-2029



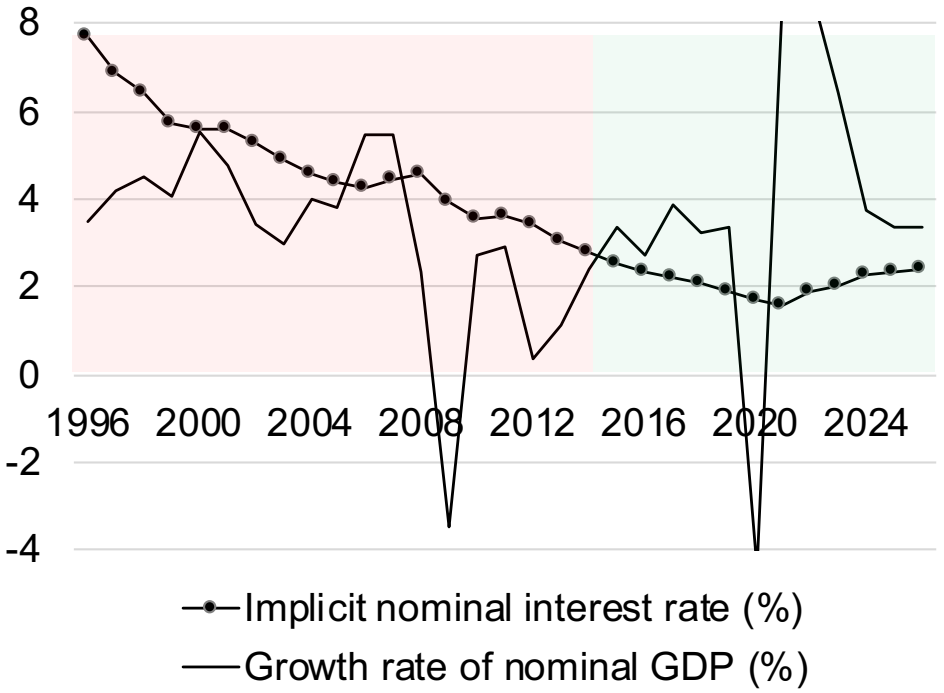
Data sources: Data until 2021 are based on OECD (Economic Outlook, nov. 2022).
Data from 2022: averages of available data and projections from IMF (WEO, Oct. 2024), OECD (Economic Outlook, dec. 2024) and Federal Planning Bureau (Economische Vooruitzichten 2024-2029, juni 2024)

1. Public debt dynamics: $r - g$, first look at data

Germany (1996-2026)



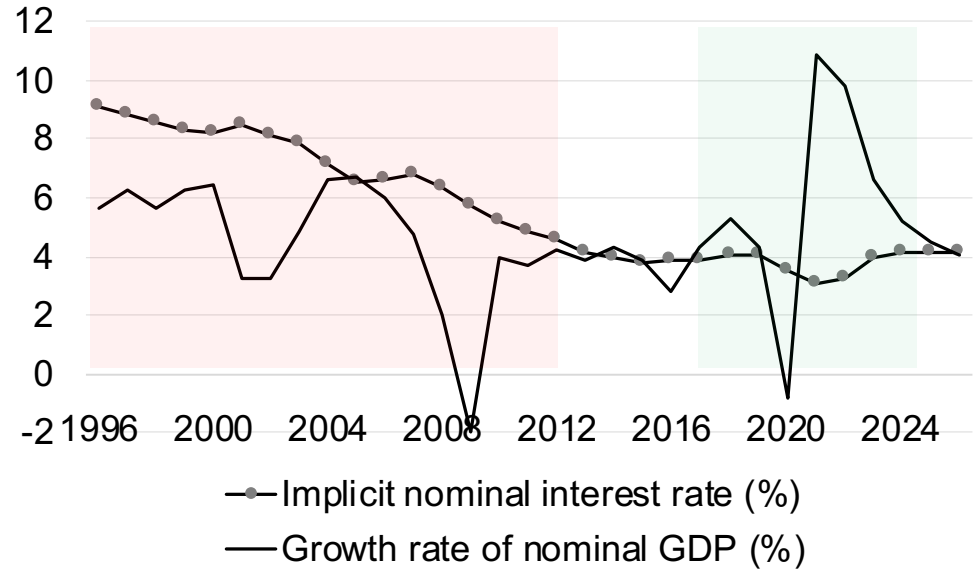
Euro area (1996-2026)



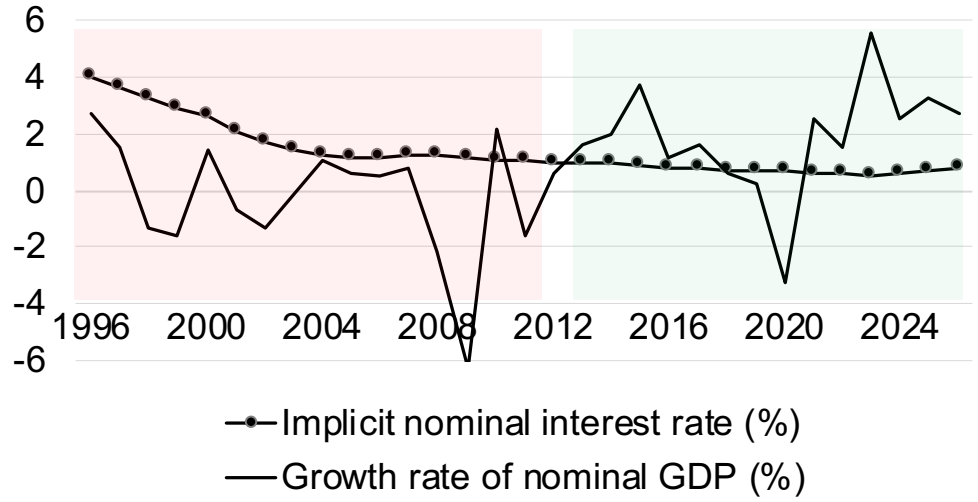
Data sources: OECD (Economic Outlook, dec. 2024)

1. Public debt dynamics: $r - g$, first look at data

US
(1996-2026)

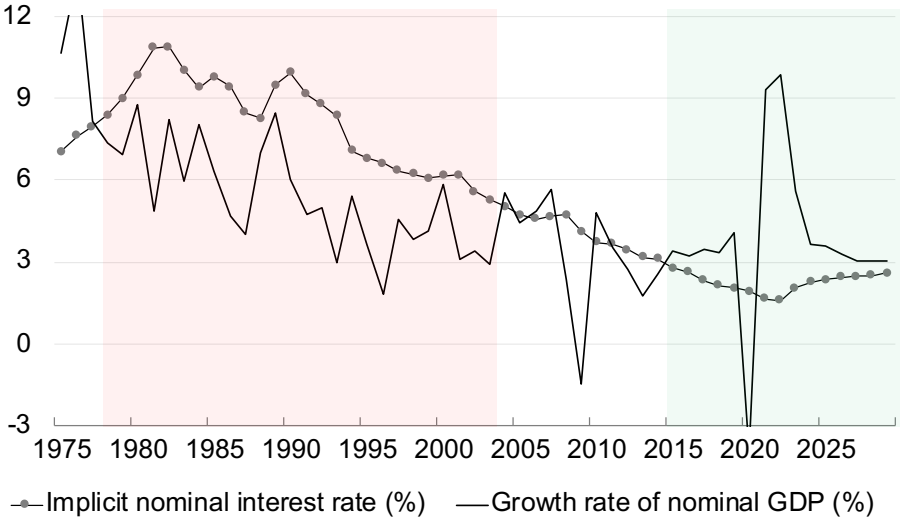


Japan
(1996-2026)



1. Public debt dynamics: $r - g$, important qualifications

Belgium: 1975-2029



Two major issues:

- The current and expected primary deficit (> 3% of GDP) is much larger than allowed for public debt stability (allowed primary deficit in 2025: about 1% of GDP, and declining).
- market rates on long-term government bonds are currently (much) higher than r. For new loans the market interest rate might even exceed g.

	2024-2025	2026-2029
g	3,6%	3,1%
r	2,3%	2,5%
market rate	2,9%	3,0%

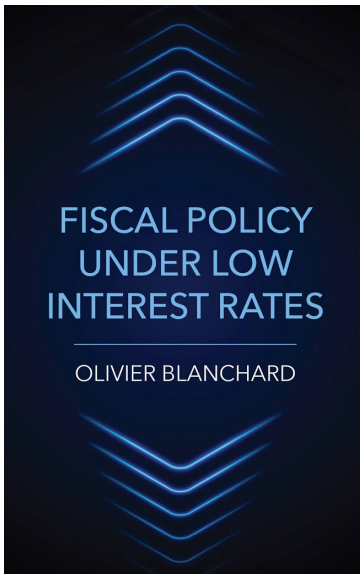
Data sources: see slide 6.

What about the future?
Will $r - g$ soon be positive again?

1. Public debt dynamics: $r - g$, what about the future ?

Most of the action was/is in the interest rate...
What to expect after the pandemic and the period of monetary contraction?
Structural changes? Or only temporary?
Different opinions.

Olivier Blanchard (2023, p. 140): ”The underlying factors behind the steady decrease in real rates over the last 30 years are still present, and suggest a likely return to sustained low rates afterward”.



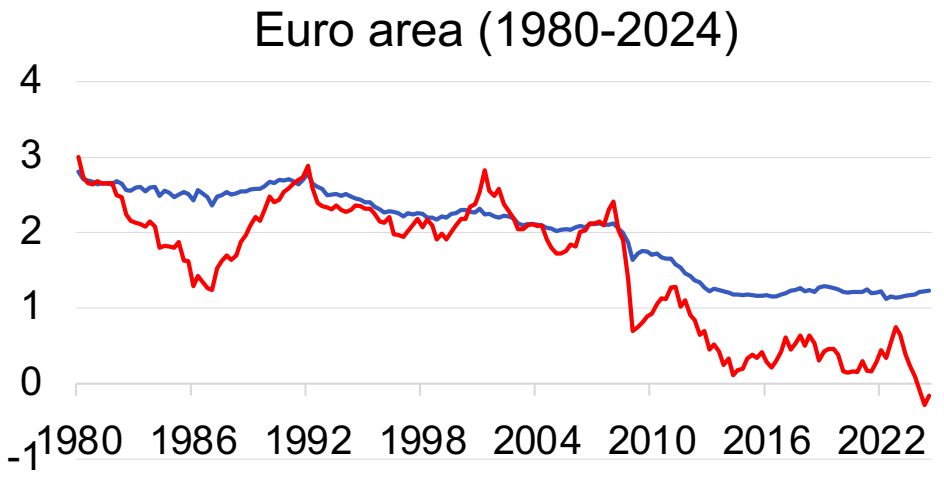
IMF (April 2023a, p. 45): ”Our analysis suggests that once the current inflationary episode has passed, interest rates are likely to revert toward pre-pandemic levels in advanced economies... This may ease the pressure on fiscal authorities over the long tem, but fiscal adjustment will still be needed in many countries to stabilize or reduced debt-to-gdp ration”.

1. Public debt dynamics: $r - g$, what about the future ?

Holston-Laubach-Williams (New York Fed, Nov. 2024) measure trend GDP growth and the neutral real interest rate:

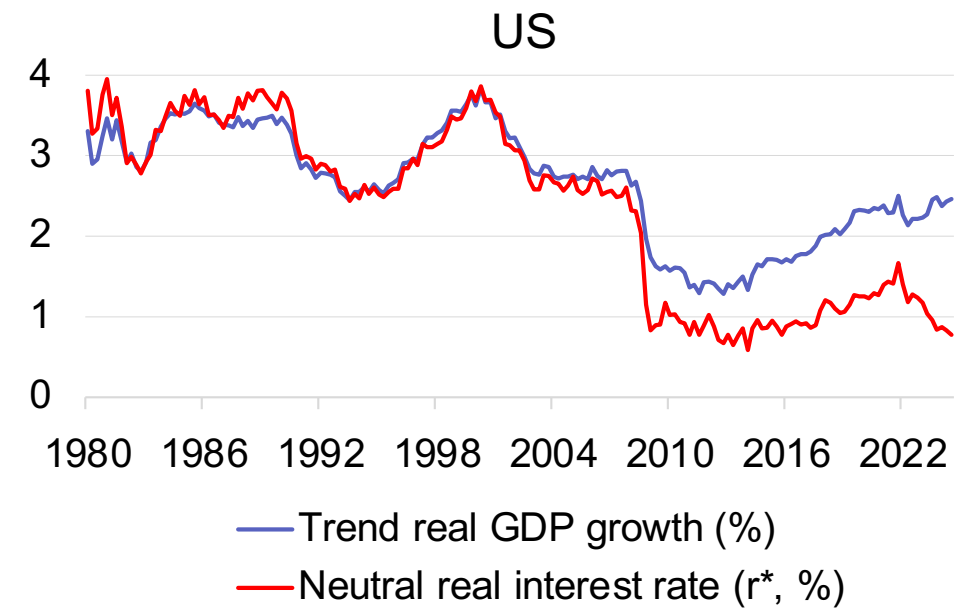
r* is still on pre-pandemic levels →

(source: https://www.newyorkfed.org/research/policy/rstar)



Christensen & Mouabbi (San Francisco Fed, March 2024) measure the neutral real interest rate for the euro area from inflation-indexed bonds:

"... the natural rate in the euro area has fallen about 2 %-points since 2002 and remains negative at the end of our sample (end 2022)".



1. Public debt dynamics: $r - g$, what about the future ?

Martin Wolf (Financial Times, 1 October 2024)

"Have real rates at last made an enduring upward jump, after their secular decline to extraordinarily low levels?"

- interest rates on inflation-protected securities have risen
- fiscal positions are stretched
- we moved from ageing to aged societies
- spending on defence and the energy transition

EU Commission's S2 Fiscal sustainability indicator / 2024 Ageing Report

"Over the long term, a progressive normalisation of financing conditions is assumed, with the r-g differential stabilising at around +1 pp. for the EU"

The Economist (20 November 2024): "Interest rates will come down sharply in 2025"

Sustainability of public debt : the impact of real long-run determinants of r – g

Content

1. Public debt dynamics and sustainability: $r - g$ as a key determinant
   - main mechanisms and a first look at data
   - important qualifications and 'what about the future?'
2. Theory and empirical research on $r - g$
   - Theory
     - real long-run determinants of r and g
     - nominal and short-run determinants
     - other unobserved or hard to quantify (common) factors
   - Empirical research for a panel of 17 OECD countries in 1981-2018
3. Projections for future $r - g$ and related debt dynamics (Belgium, other EU)
4. Conclusions, discussion

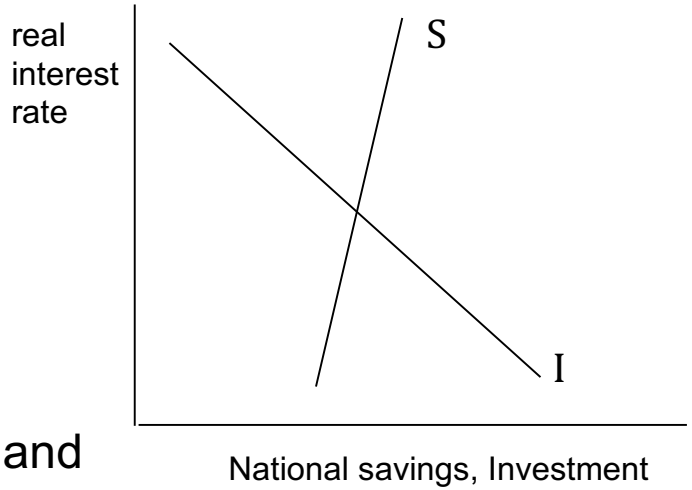
2. Theory and empirical reserach on $r - g$

(Heylen, Mareels and Van Langenhove, JIMF, 2024)

Theory: real long-run determinants

Effect on $r - g$

- Economic growth (cf. neoclassical and endogenous growth theory)
  - Rate of technical progress (TFP-growth) : -
  - Growth rate of (employed) population : -

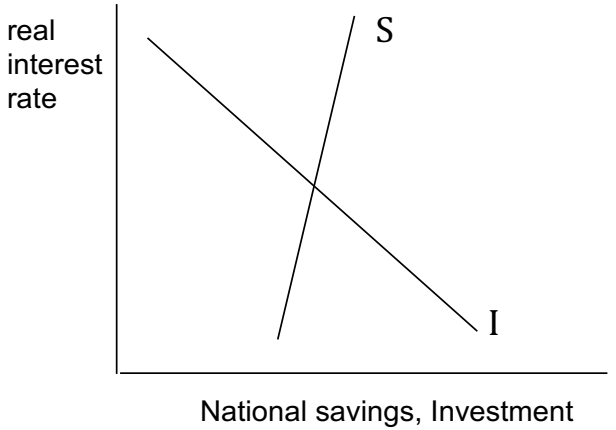


- Real interest rate

  ("natural / neutral rate of interest", r^*)

  Via their impact on the productivity of capital and private propensity to invest:
  - Rate of technical progress (TFP-growth) : +
  - Growth rate of (employed) population : +

2. Theory and empirical reserach on $r - g$



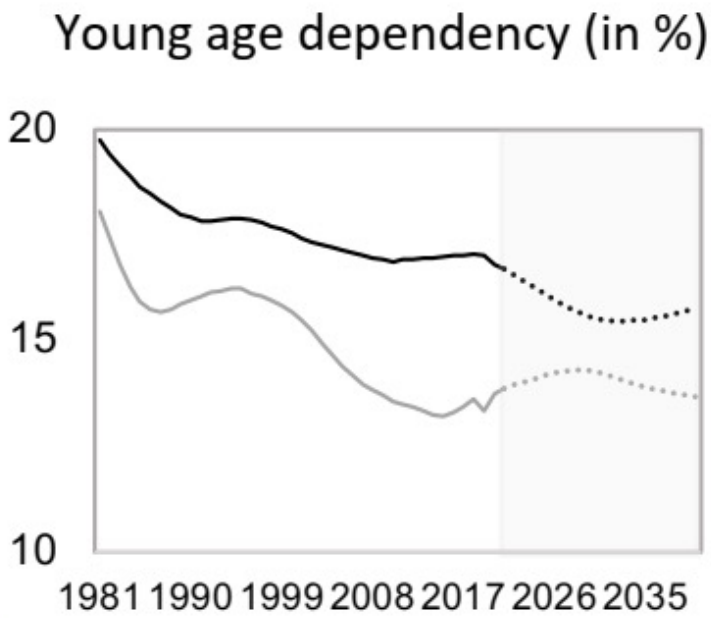
Theory: real long-run determinants

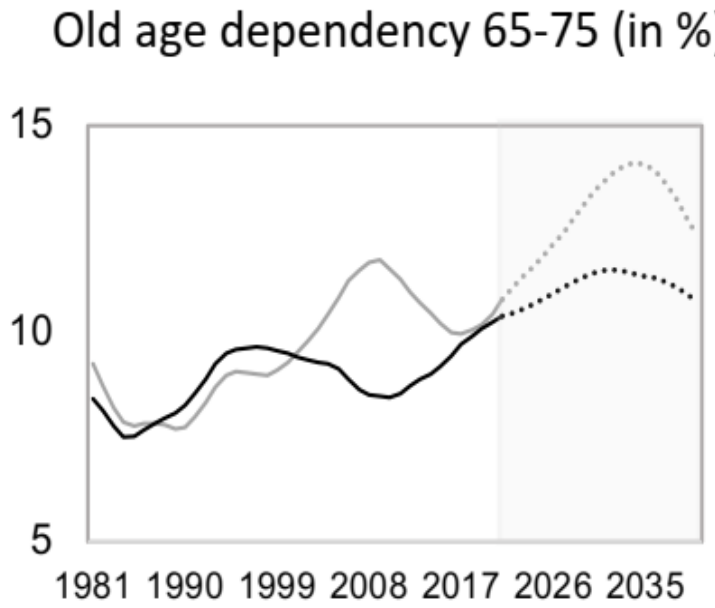
- Real interest rate — Effect on $r - g$

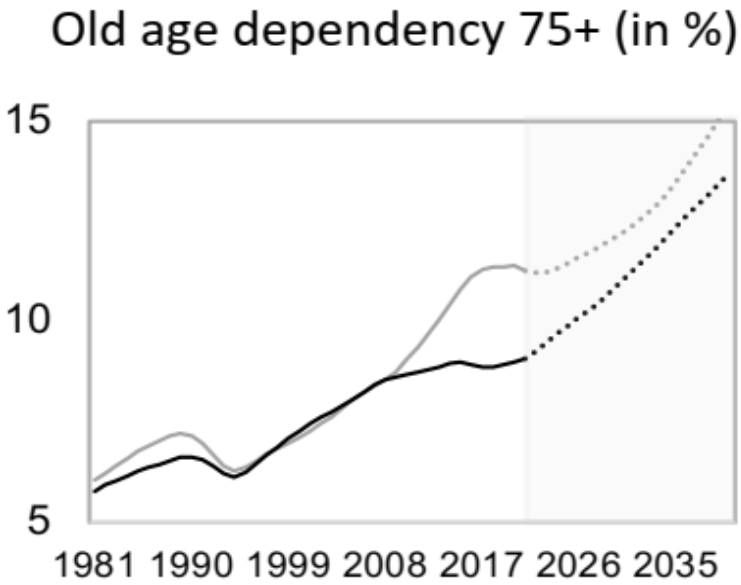
Via their impact on savings:

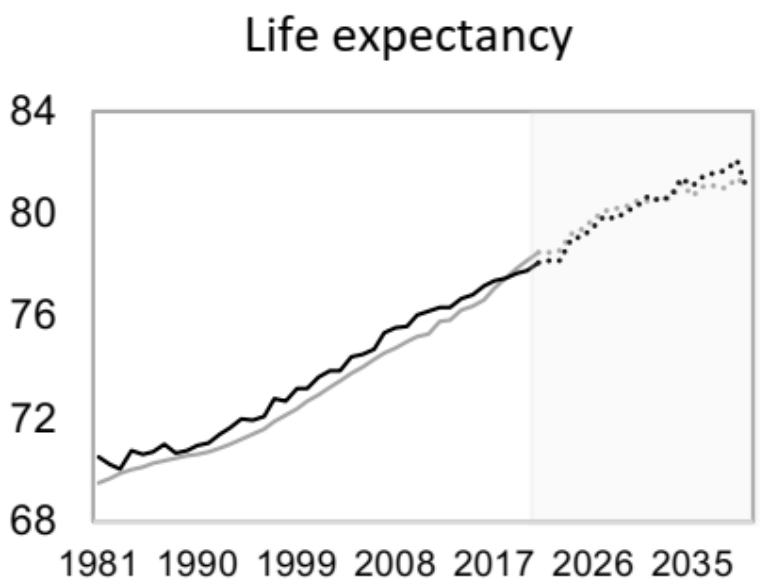
▪ Life expectancy	-
▪ Demographic structure	
- Fraction of population 0 – 14	+ / ?
- Fraction of population 65+	+ / ?
* fraction 65 – 75	?
* fraction 75+	+
▪ Inequality (gini disposable income)	-
▪ Fiscal policy	
- primary deficit (% of GDP)	+
- public debt (% of GDP)	+

Data : illustration Belgium, Germany







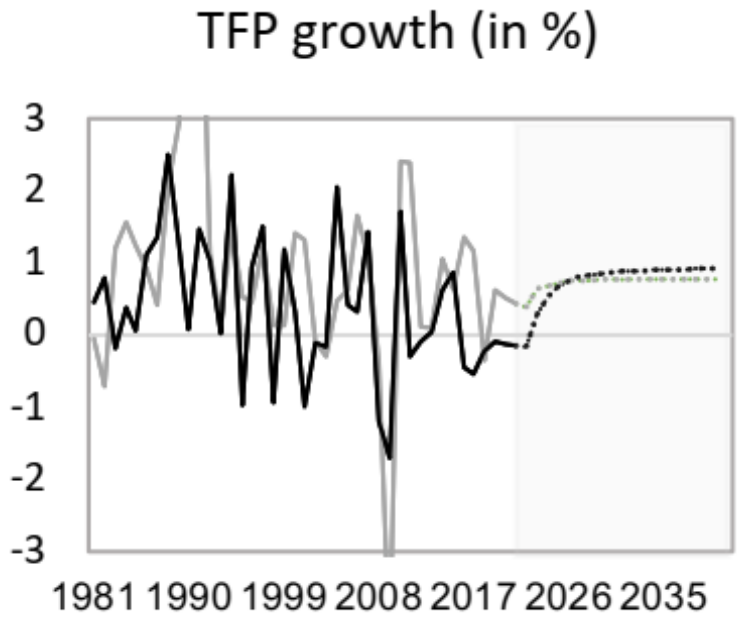


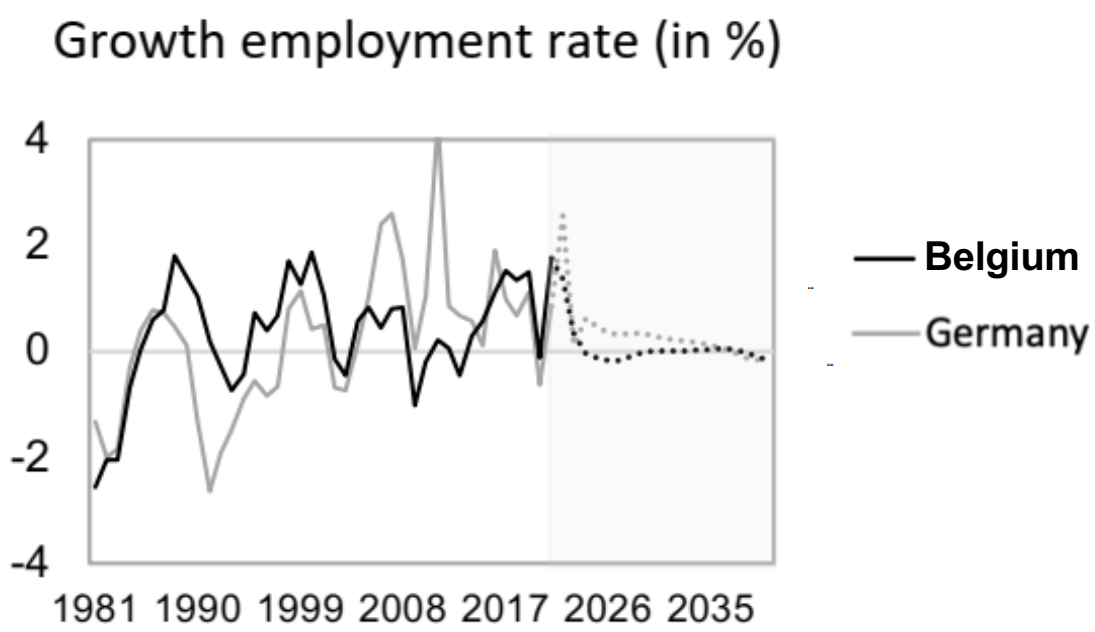
Belgium

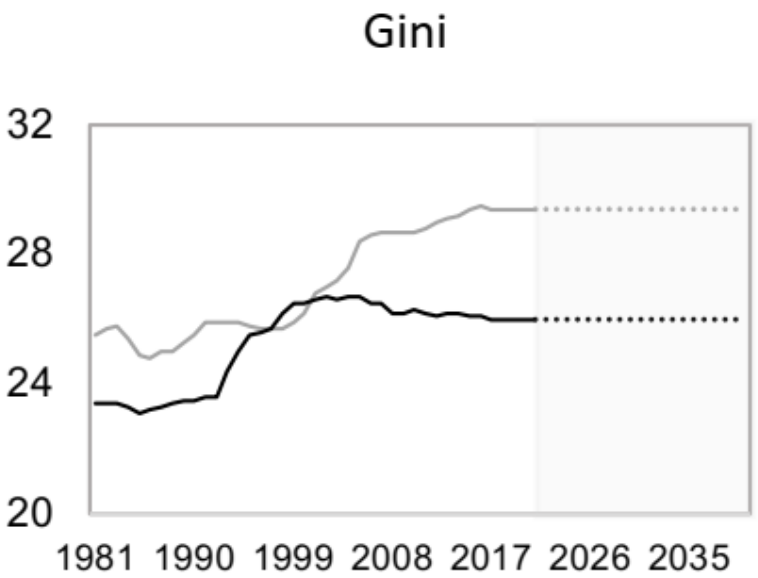
Germany

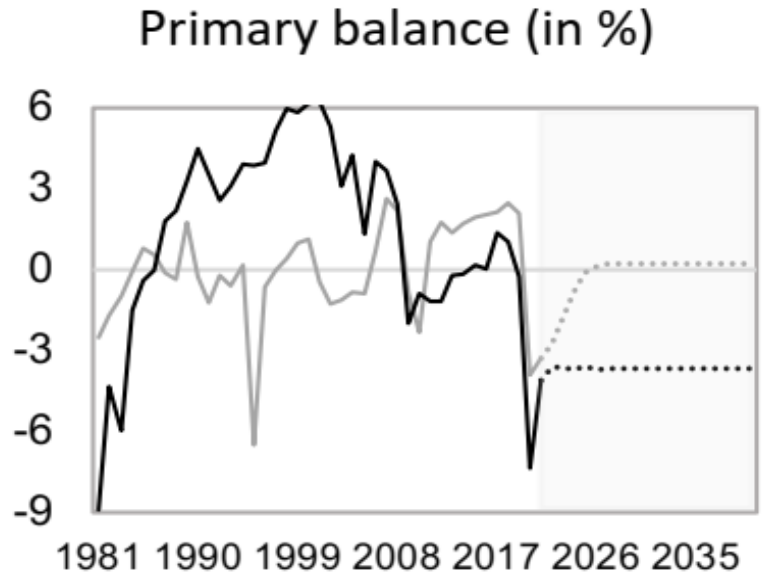
Note: the grey areas contain the projections on the future of these variables (cf. later model simuation)

Data : illustration Belgium, Germany









2. Theory and empirical reserach on $r - g$

Theory: nominal and short-term determinants of growth and interest rate

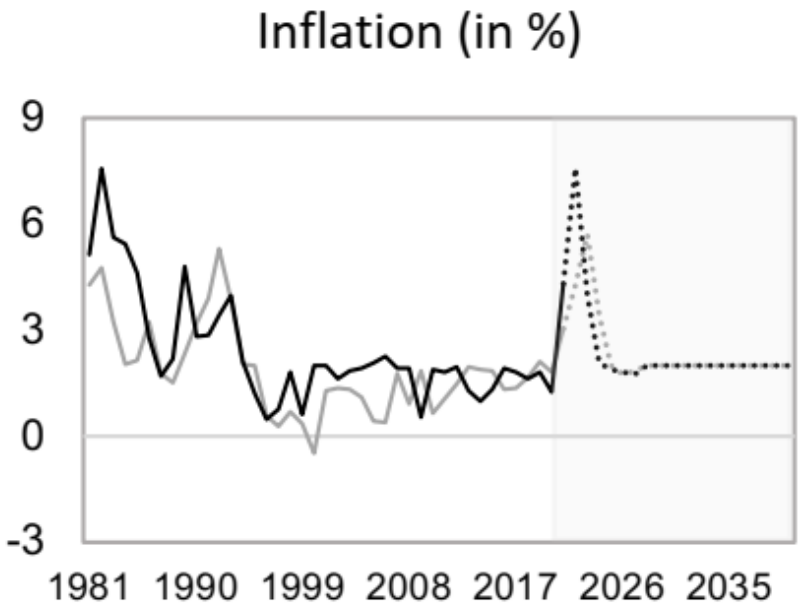
	Effect on $r - g$
▪ Monetary policy	
- short-term interest rate	+
- Purchase of government bonds (QE)	-
▪ Inflation (immediate effect on g, slow on r)	-
▪ business cycle (output gap)	-

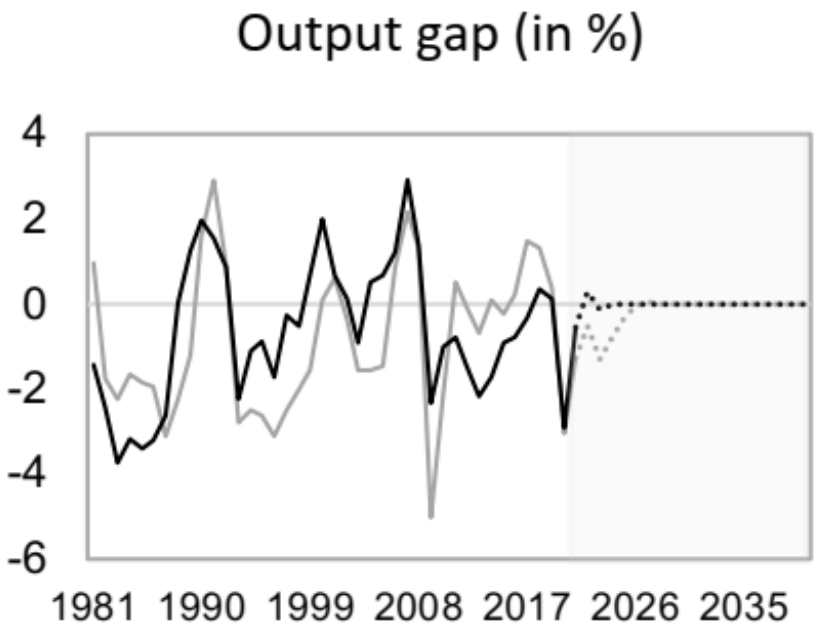
Other (common) factors

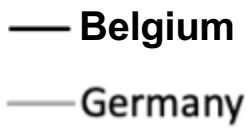
(most of which are hard to observe, and may affect countries differently → empirical challenge)

- global trend to financial liberalization since the 1980s
- shift in risk aversion and higher demand for safe assets after the financial crisis
- increased market power of big firms
- ....
- $r - g$ in the US

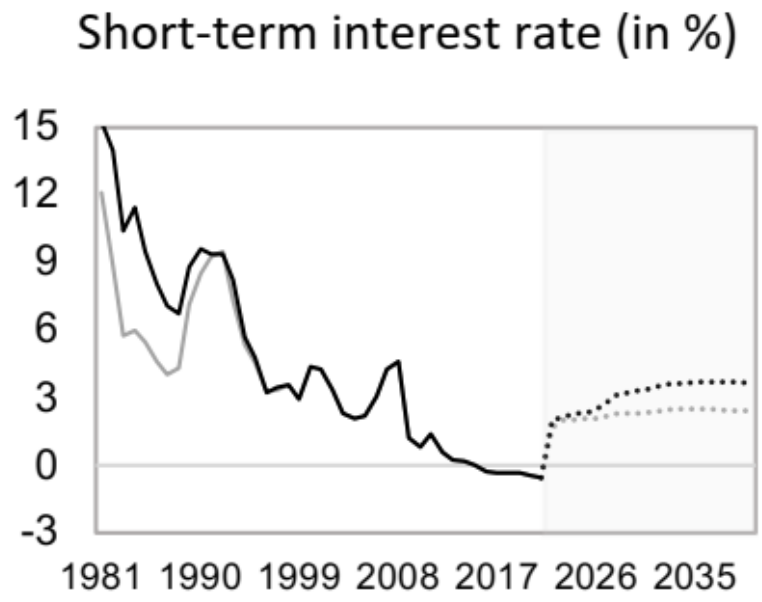
Data : illustration Belgium, Germany

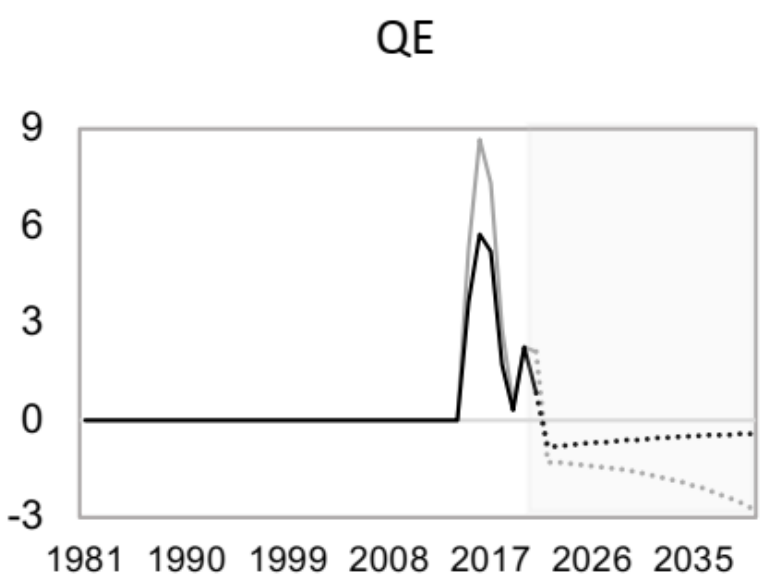






Note: the grey areas contain the projections on the future of these variables (cf. later model simuation)





2. Theory and empirical reserach on $r - g$

(Heylen, Mareels and Van Langenhove, JIMF, 2024)

Estimated equation

$$(r-g)_{i,t} = \alpha_i + \sum_{j=1}^{K} \beta_j X_{i,t}^{j} + \nu_{i,t}$$

$$\nu_{i,t} = \lambda_i f_t + \varepsilon_{i,t}$$

i = 1 – 17 countries
t = 1981 – 2018

$(r-g)_{i,t}$: $r-g$ in country i and year t

α_i : country-specific fixed effect

$X_{i,t}^{j}$: value of explanatory variable X^j in country i and year t

β_j : coefficient on variable X^j, to be estimated

$\nu_{i,t}$: error term (residual) of the regression in country i and year t (may capture an unobserved, potentially non-stationary common factor)

λ_i : country-specific factor loading on the common factor f_t

$\varepsilon_{i,t}$: white noise error term

2. Theory and empirical reserach on $r - g$

(Heylen, Mareels and Van Langenhove, JIMF, 2024)

Explanatory variables X^j

Variable	*Definition*
TFP growth	yearly growth rate of TFP (in %)
working age population growth	yearly growth rate of population aged 15-64 (in %)
employment growth	yearly growth rate of employment (persons, in %)
old age dependency	population 65 and older in % of total population
young age dependency	population aged 0-14 in % of total population
life expectancy	life expectancy at birth at time t-20 (in years)
Gini index	Gini index for disposable income (scale 0-100 with 0 perfect equality)
inflation	yearly change of the GDP deflator (in %)
output gap	(actual output – potential output) / potential output (in %)
short-term interest rate	3-month government T-bill rate (in %)
primary balance	primary balance of general government (in % of GDP)
public debt ratio	gross government debt (in % of GDP)
QE	public sector assets bought by the central bank (flow) in % of outstanding public debt

Estimation results : explanation of $(r-g)_{i,t}$ in 17 countries in 1981-2018

	(1)	(3)	(6)
TFP growth	-0.956***	-0.929***	-0.863***
	(0.062)	(0.050)	(0.055)
Working age population growth	0.176	0.429*	
	(0.358)	(0.256)	
Employment growth		-0.598***	
		(0.040)	
Growth of the employment rate [2]			-0.515***
			(0.037)
Old age dependency	0.004		
	(0.105)		
Old age dependency 65-75		-0.018	0.016
		(0.102)	(0.091)
Old age dependency 75+		0.387**	0.131
		(0.151)	(0.150)
Young age dependency	0.064	-0.073	-0.046
	(0.082)	(0.056)	(0.075)
Life expectancy	-0.274**	-0.633***	-0.483***
	(0.123)	(0.112)	(0.120)

Notes : 1. Estimated standard errors in brackets. * $p<0.1$; ** $p<0.05$; *** $p<0.01$. (Driscoll-Kraay corrected).
2. Difference between the growth rate of employment and the growth rate of working age population

Estimation results : explanation of $(r-g)_{i,t}$ in 17 countries in 1981-2018 (continued)

	(1)	(3)	(6)
Gini	-0.343***	-0.247***	-0.209***
	(0.111)	(0.065)	(0.040)
Primary balance	-0.137***	-0.037	-0.078**
	(0.038)	(0.025)	(0.031)
Public debt ratio	0.027***	0.031***	0.019***
	(0.009)	(0.008)	(0.005)
Public debt ratio x Dummy90+		-0.00004	0.005***
		(0.003)	(0.002)
QE		-0.048*	-0.040**
		(0.025)	(0.020)
Inflation	-1.104***	-1.068***	-0.990***
	(0.054)	(0.042)	(0.039)
Output gap	-0.143***	-0.054**	-0.055**
	(0.038)	(0.024)	(0.027)
Short-term interest rate	0.453***	0.393***	0.337***
	(0.041)	(0.025)	(0.029)
Observations	643	643	643
Country fixed effects	yes	yes	yes
Cross-sectional average incl. (*)	yes	yes	yes
Cointegration	yes	yes	yes
R^2 adjusted	0.84	0.89	0.92

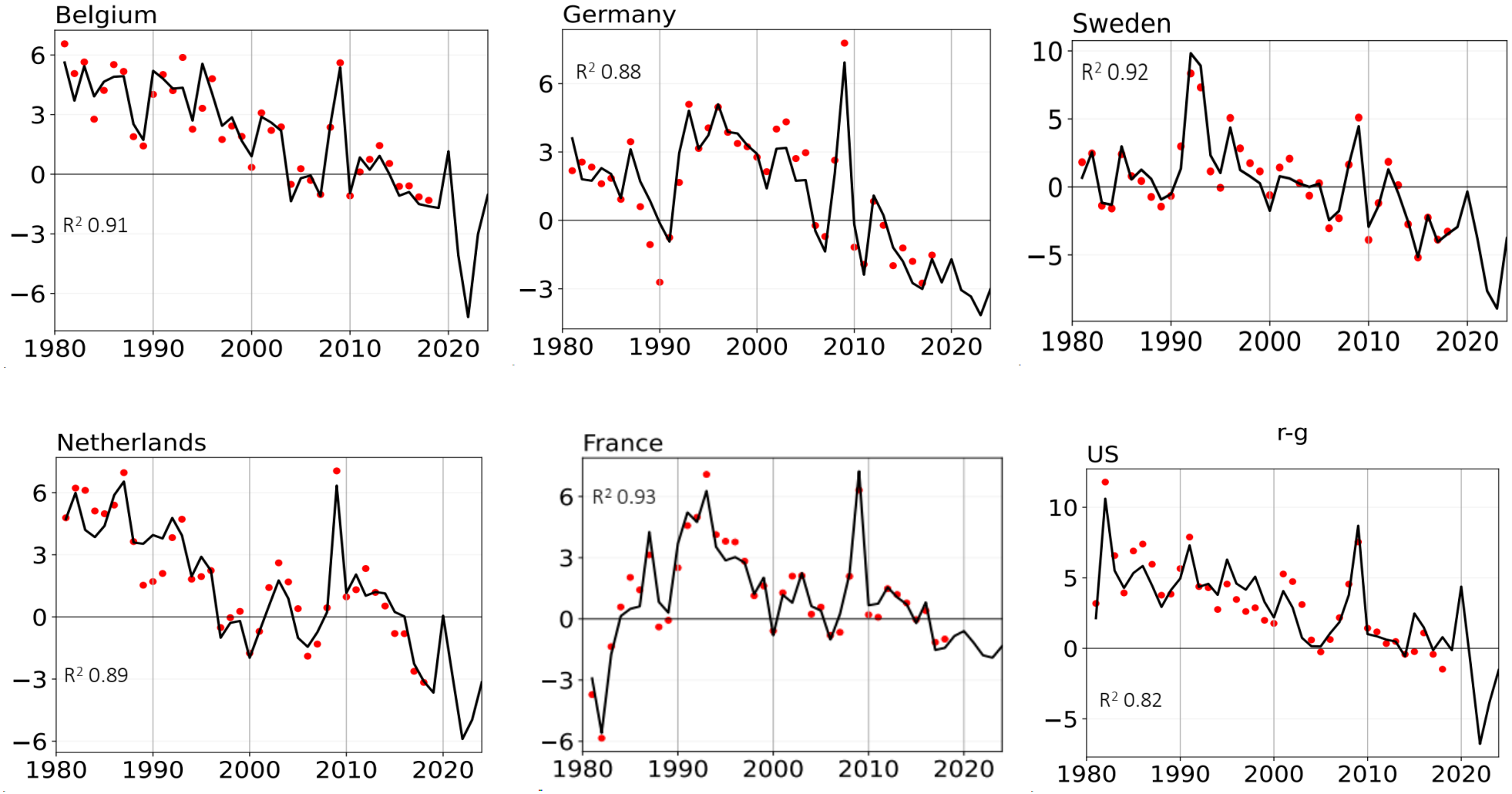
Note:
(*) In each column we also include the cross-sectional average of $(r-g)_{i,t}$. In column (1) all countries have the same coefficient on this CSA (λ =0.27), in columns (3) and (6) each country has its own coefficient λ_i.
In column (6) we additionally add $(r-g)_{US,t}$ as explanatory variable, with each country again getting its own coefficient on this common variable. The average of these coefficients is only 0,06, but statistically signficant.

2. Theory and empirical reserach on $r - g$

Panel data study, 17 countries, 1981-2018
Main explanatory variables

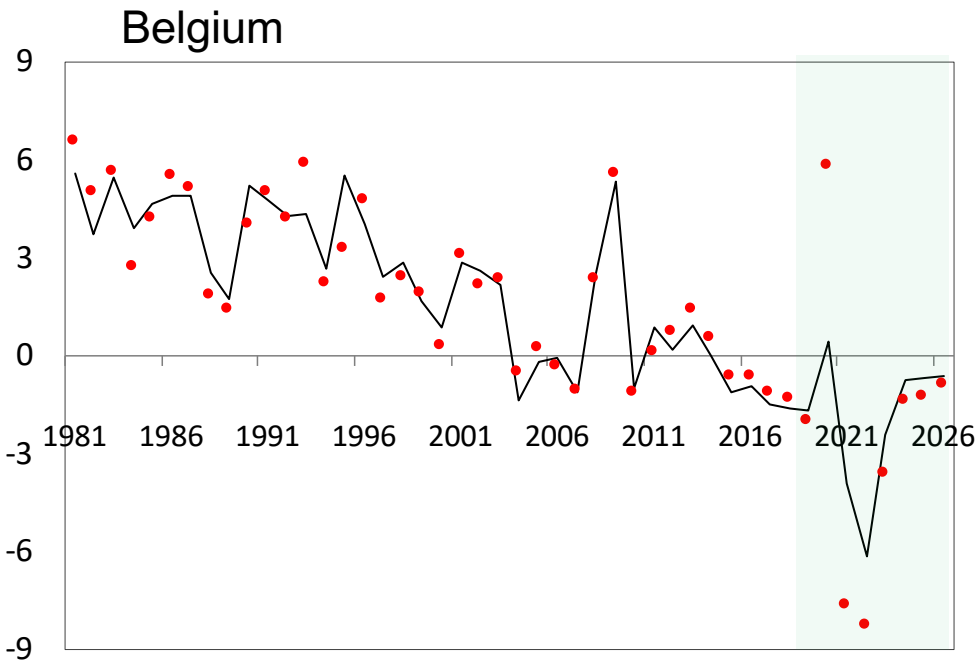
	Theoretical effect on:		**Empirical result**
Explanatory variable:	**g**	**r**	**r - g**
Rate of technical progress	+	+	-
Growth of the employment rate	+	+	-
Life expectancy		-	-
Population (65-75)/total		?	?
Population (75+)/total		+	+
Population (0-14)/total		?	?
Inequality (Gini disposable income)	-/+	-	-
Primary deficit		+	+
Public debt ratio		+	+ worse when initial debt is > 90%
Other: Monetary policy, inflation, r-g in US,...			confirmation of expected effects

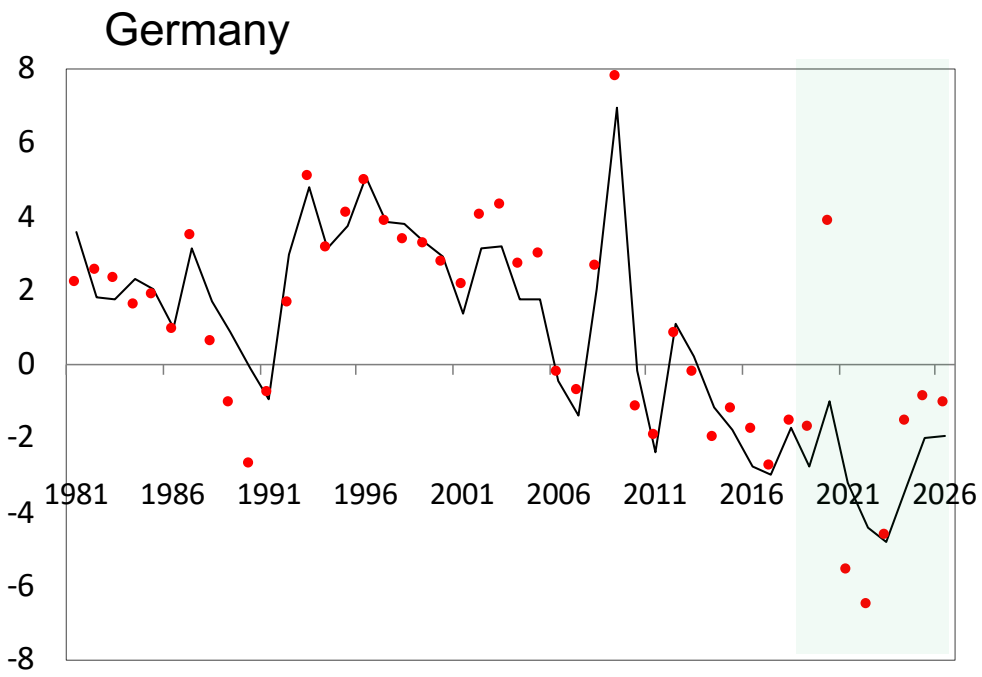
Quality of the estimated model for $r - g$ in 1981-2018



R^2 : explanatory power of the estimated model in 1981-2018

Quality of the estimated model for $r - g$ in 1981-2026
(out of sample test, 2019-26)





Data — Model (and model projection for 2019-2026)

Sources for data in 2022-2026: see slides 6 and 7.

Sustainability of public debt : the impact of real long-run determinants of r – g

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1. Public debt dynamics and sustainability: $r - g$ as a key determinant
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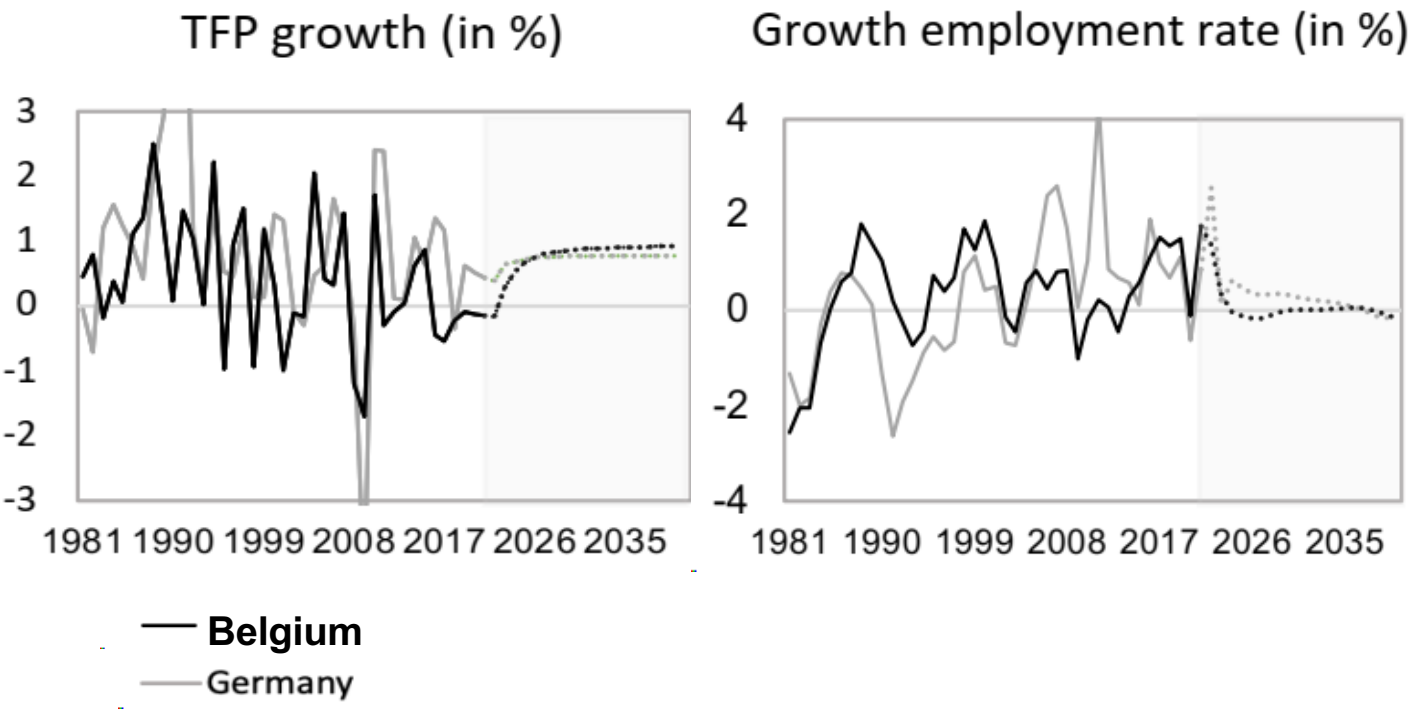
3. Projections for future $r - g$ and related debt dynamics

Imposed data or adopted assumptions regarding the future evolution of the determinants of $r - g$ (see grey areas on the figures on slides 17-18 and 20) :

→ Demographic variables: demographic projections of OECD and World Bank

→ TFP growth and growth of the employment rate :

OECD: Long-term baseline projections (2021)



2030-2040	Average annual Growth TFP-growth	Empl. Rate
SCvV	1,1%	0,3%
OECD	0,9%	0,0%
EU 2024 Ageing Report	0.7%	0.2%

Note:
SCvV = Studiecommissie voor de Vergrijzing 2022 (Belgian ageing study comm)

3. Projections for future $r - g$ and related debt dynamics

Imposed data or adopted assumptions regarding the future evolution of the determinants of $r - g$:

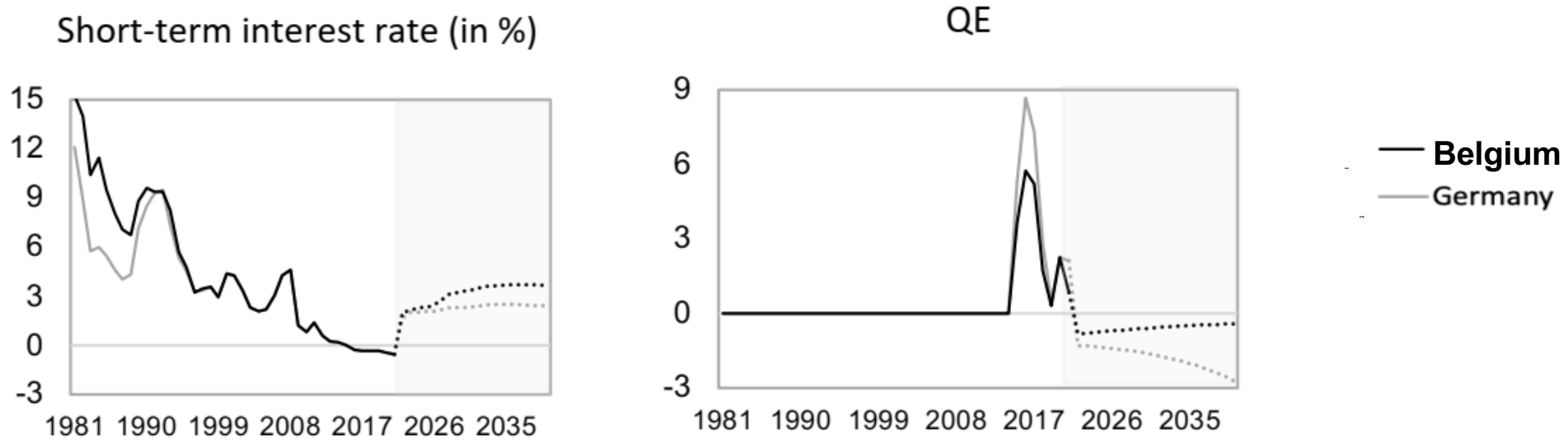
→ Inequality (Gini) : assumed constant at the level of 2018

→ Inflation: IMF projections until 2027, afterwards: assumed 2%

→ Output gap: IMF projections until 2027, afterwards: assumed 0%

→ Short-term interest: application of pure expectations theory

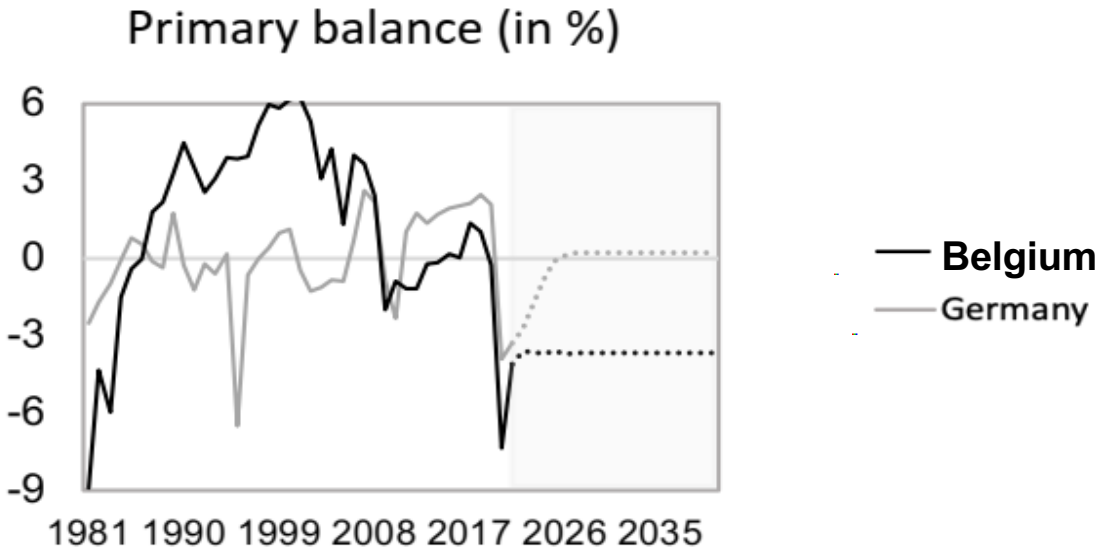
→ QE: all assets bought in previous years redeemed by 2040 (= stock of government bonds held by the central bank is gradually brought to zero by 2040).



3. Projections for future $r - g$ and related debt dynamics

Imposed data or adopted assumptions regarding the future evolution of the determinants of $r - g$:

→ Primary balance: IMF (WEO, October 2022) projections until 2027
From 2028 we assume the primary balance constant at the 2027 level)

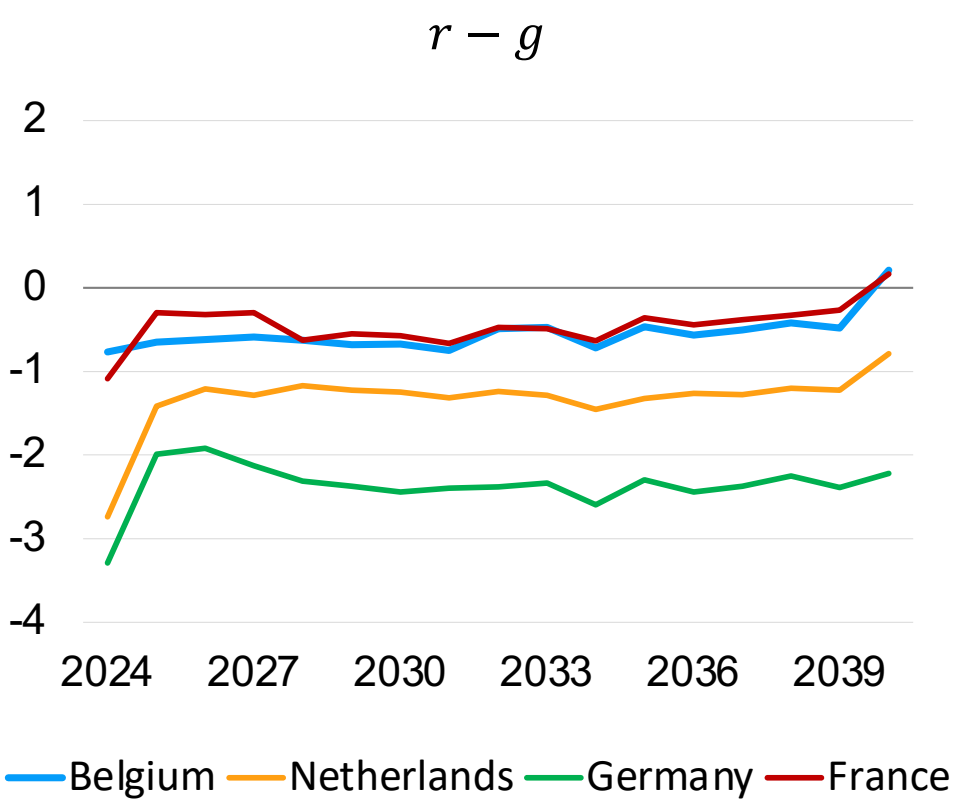


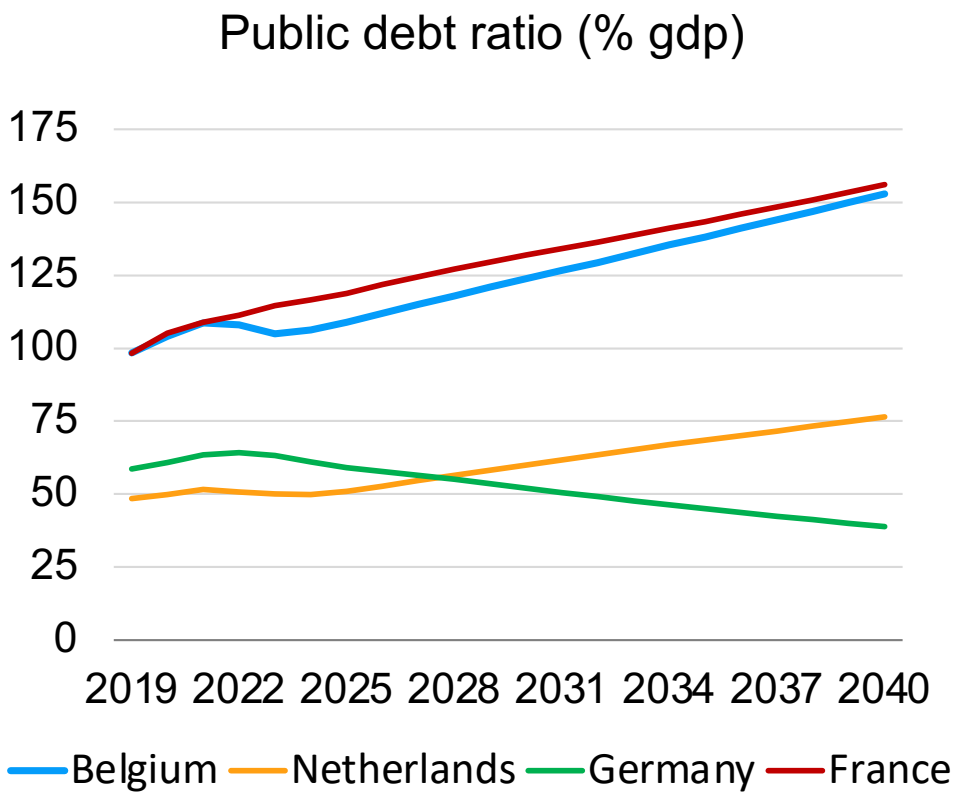
→ Public debt ratio: endogenous. Possibility for countries to end up in a vicious / virtuous circle.

→ Impact of future unobserved common factors and developments in the US is set to zero. Focus on domestic drivers.

3. Projections for future $r - g$ and related debt dynamics

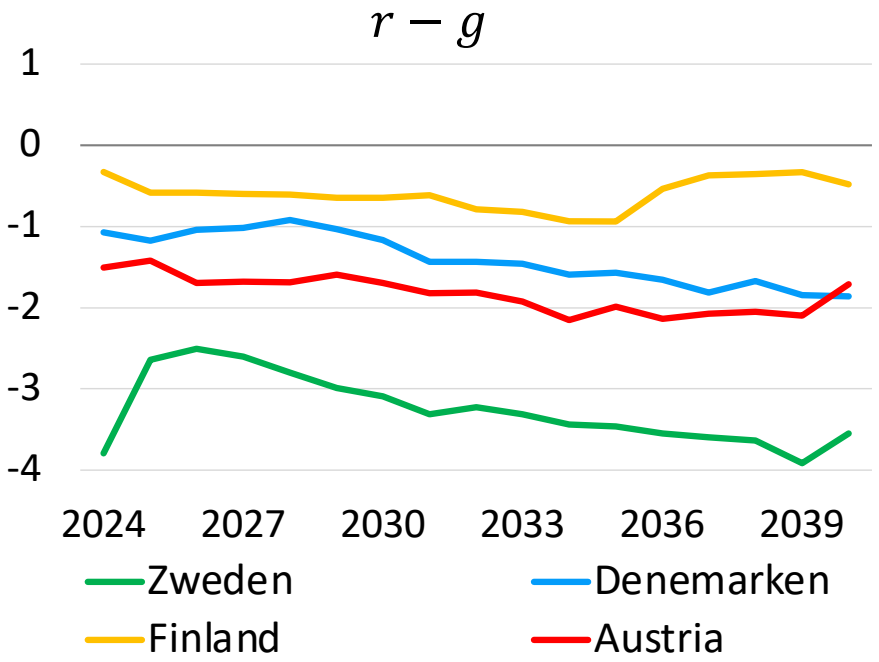
Baseline projections 2024-2040

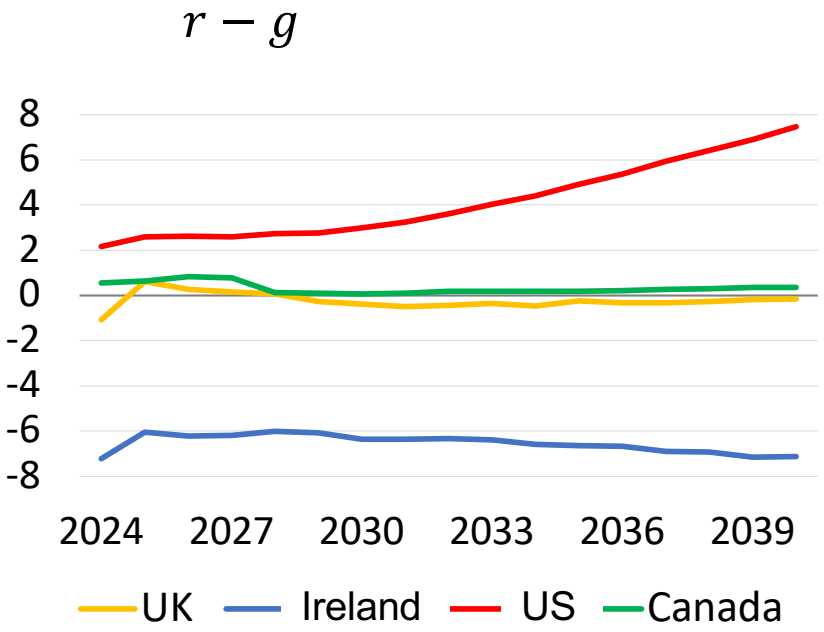


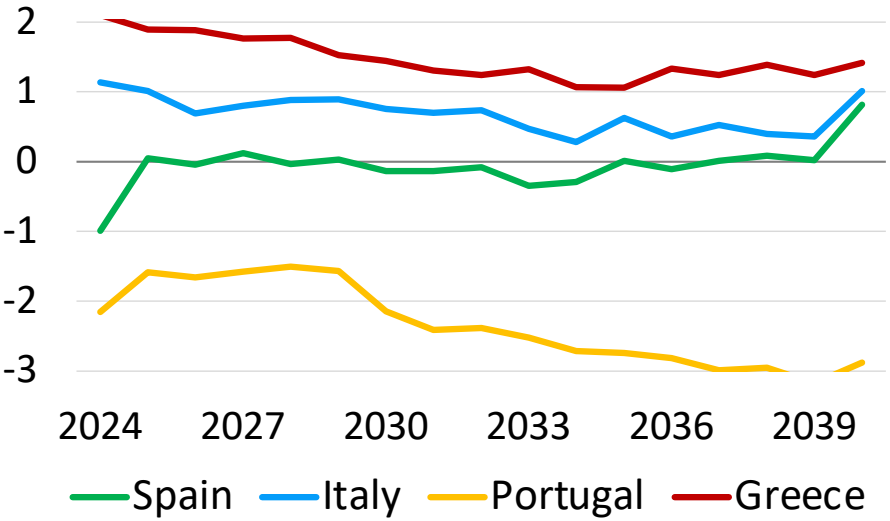


Note: Assumed primary balances from 2027 (IMF, WEO, October 2022):
Belgium: -3,7%; Netherlands: -2,5%; Germany: 0,2%; France: -3,1%

3. Projections for future $r - g$ and related debt dynamics





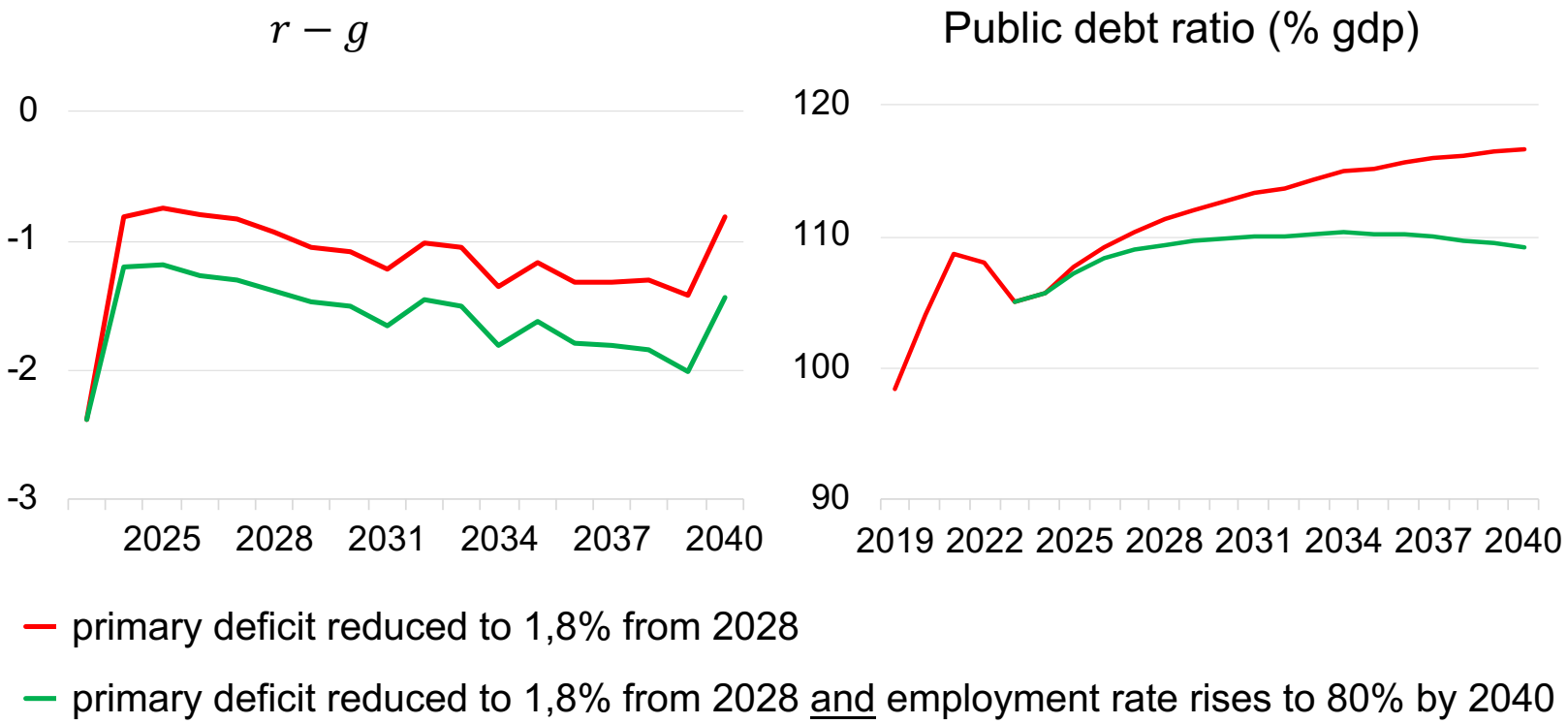


More details: see Heylen et al. (2024)

All countries shown here are expected by the IMF (WEO, October 2022) to have either a small primary surplus from 2027, or a small deficit (smaller than 1% of GDP). The three exceptions among the countries shown here are Spain (deficit of 2%), Finland (deficit of 2.6%) and most problematic the US (deficit of 3.5% → vicious circle of rising r-g and exploding debt).

Alternative simulations

→ Policy matters (e.g. Belgium): from an exploding to a sustainable debt ratio.



But achieving this will be hard.

→ Other scenarios, e.g. faster technical progress (see Heylen et al., 2024).

→ Shocks (happen all the time).... If the shocks of 1982-2018 happened again, it would raise the projected r-g in a majority of scenarios by about 0,5 to 1%-point.

Sustainability of public debt : the impact of real long-run determinants of r – g

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4. Conclusions, discussion

- Empirical study of the drivers of $r - g$ in 17 OECD countries. Focus on role of real long-run drivers (technical progress, employment growth, life expectancy and demographic structure, inequality, fiscal policy)
- Baseline expectation : $r - g$ may remain negative in the next two decades in most EU countries that we studied. (Not in the US, and surely not immune to uncertainty).
- The debt-carrying capacity of governments is substantially higher now (than in the 1980s or 1990s when $r > g$). Good news, but not a free-for-all for debt accumulation.
- Public debt stability is consistent with having a (limited) primary deficit.
- Belgium's primary deficit is far too high.
- Policy matters! One simulation : a reduction of this deficit to 1,8% of GDP, combined with effective employment policies, may bring $r - g$ below -1% and allow a stable debt ratio of about 110%. The same holds for combinations with other growth policies.
- Endogeneity of $r - g$ is key.

4. Conclusions, discussion

- (Revision of) European fiscal rules and governance:
  - the increased attention to debt sustainability and focus on a medium-term path for 'net primary expenditures' are clearly positive, but....
  - why see public debt almost by definition as a burden on future generations?
    why still see 60% as the optimal to be achieved target?
    why assume that $r - g$ will tend to +1%? (Is this consistent with the 60%?)
  - with $r < g$, and a high excess of private savings over private investment in the euro area (6% of GDP), why not use public debt as an opportunity to ease future burdens?
    - infrastructure, R&D,... (content of expenditures)
    - public debt as part of optimal, welfare increasing climate policies to avoid the much higher burden of future climate damage (IMF, 2023b; Kotlikoff et al., 2021).

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Heylen, F., Mareels, M. & C. Van Langenhove (2024), Long-run perspectives on $r - g$ in OECD countries: An empirical analysis, *Journal of International Money and Finance*, https://doi.org/10.1016/j.jimonfin.2024.103093

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Kotlikoff, L., Kubler, F., e.a. (2021) Making carbon taxation a generational win win, *International Economic Review,* 62, 3-46.